An Extension to Rough Terrains of the MPC/CLF Mobile Vehicle Navigation Approach

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***Abstract*— Model Predictive Control (MPC) combined with the Control Lyapunov Function (CLF) optimization framework has been used for the navigation planning of indoor mobile robots moving in flat terrains. This approach guarantees the stability in the Lyapunov sense of the planned trajectory, provided that a proper navigation function is included in the CLF. In this paper, an extension of the combined MPC and CLF approach is proposed for navigation planning in outdoor rough terrains. The extension is based on a novel theoretical consideration. The proposed algorithm ensures obstacle avoidance as well as the selection of an appropriately traversable terrain by optimizing an objective function which considers the terrain roughness level along admissible paths.**

I. INTRODUCTION

Planetary explorations, search and rescue missions in hazard areas, surveillance, humanitarian de-mining, as well as agriculture applications such as pruning vine and fruit trees, represent possible fields of using autonomous vehicles in natural environments. The unstructured environment and the terrain roughness including dynamic obstacles and poorly traversable terrains pose a challenging problem for the autonomy of the vehicle.

A nice overview of motion planning has been presented in [1]. The main focus of the early research stage was finding collision-free paths. In [2] the potential field approach for real-time obstacle avoidance was introduced while the concept of navigation functions was illustrated in [3]. The following work given in [4] included the general path planning problem using high d.o.f. manipulators. Also, the motion planning for mobile robots operating in a structured environment was discussed, dealing with local minima problem as well. Ge and Cui dealt with the problem of moving obstacles using the potential field method [5].

The research on motion planning evolved by adding the capability of taking into account the vehicle motion constraint within the well known dynamic window approach [6], [7]. This subject was extended to the high-speed navigation of a mobile robot in [8] by the global dynamic window approach, as the generalization of the dynamic window approach. A combination of the dynamic window approach with other methods yielded some improvements in long-term real-world applications [9]. Dubowski and Iagnemma extended the dynamic window approach to rough terrains introducing the vehicle curvature-velocity space. In this space the stability constraints of the vehicle, for instance expressed by limit values of the roll-over and side slip indexes, can be easily described. The given algorithm was also suitable for high speed vehicles and appropriate for real-time implementation [10]–[12].

The work presented in this paper was mainly inspired by the MPC/CLF framework (Model Predictive Control and Control Lyapunov Function) derived and explained in [13] and its application to the mobile robot navigation problem in flat terrains proposed in [14]. The proposed algorithm extends and adapts the MPC/CLF optimization framework from flat to rough terrains preserving its main property of guaranteed task completion. This means taht the framework uses the MPC/CLF control paradigm for navigation purposes providing a merge of a local and a global planning within a compact single framework. This gives the possibility of proving the guaranteed task completion using the stability concept of the MPC/CLF framework since Lyapunov function consists of the navigation function that deals with global planning. The cost function that is locally minimized within the MPC horizon describes the level of roughness that should be estimated for all candidate paths. The level of roughness along a candidate path represents the information on how hard is to traverse this path by the mobile vehicle. The presented MPC/CLF scheme for rough terrains navigates the vehicle to follow less rough paths unlike those generated by the MPC/CLF for flat terrains. The main practical consequence of the selection of less rough terrain sections is the increase of the vehicle ability for high-speed maneuvers that do not cause unwanted effects such as sideslip and rollover. In accordance to the MPC optimization, any additional constraint can be imposed into the MPC/CLF navigation, such as those related to vehicle stability preventing from vehicle rollover and unnecessary sideslip. Unlike sample-based approaches where the optimization is inherently off-line, such as one variant of lattice roadmap paradigm [15], [16] where state lattices where created to represent differential constraints of the vehicle, the MPC/CLF method is suitable for online operations. In this paper, the analytic proof of the maximum task completion time and path length are also presented.

In Section II, the MPC/CLF optimization scheme [13] and its application to the navigation planning for flat terrains [14] are reviewed. Section III explains theoretically the approach proposed to deal with different levels of terrain roughness, while Section IV gives analytical proof of the maximum time and length of the task completion. The simulation results and the conclusion are presented in Section V and VI.

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II. Previous work used in this paper

A. Dynamic Window Approach

The idea of using a vehicle velocity space for the local obstacle avoidance appeared in [6]. It aims at optimizing an objective function dealing with speed, safety and goal-directedness, taking into account the kinematic model of the vehicle as well as its physical velocity and acceleration limits. In order to optimize the objective function, only admissible velocities are considered, which are defined as all the velocity tuples (v, ω) that are achievable in one control cycle, for the current vehicle state, that generates collision-free trajectories, and that allow the vehicle to stop before hitting an obstacle in the successive control cycle. In addition to this work, in [7] the authors have included the motion dynamics as an additional constraint into the vehicle velocity. This framework is known as Dynamic Window Approach and its main problem was the lack of information on the connectivity of an obstacle-free terrain, hence the vehicle can get stuck in local minima before reaching the target position.

In [8] the performance index of the dynamic window approach has been modified by introducing a navigation function. [1], [3] aimed at overcoming the local minima problem by adding information about the connectivity of an obstacle-free terrain space. This approach was called Global Dynamic Window Approach, since the navigation function has only one global minimum at the goal position. Regardless of the fact that the purpose of the navigation function was to globally navigate the vehicle toward the goal, actually this result could not be achieved. In fact, an example has been found where the algorithm failed to reach the target [17], and it was not possible to prove the convergence of the algorithm, namely to guarantee the achievement of the goal.

Recently, the idea of using the curvature-velocity space has been also widely explored and successfully implemented for trajectory planning in rough terrains [10]–[12], [16] and [23].

B. MPC/CLF approach to navigation planning

The work presented here was mainly inspired by the MPC/CLF framework, described, for instance in [13], where the strong connections between optimal control for infinite time horizon, minimum norm pointwise control, Lyapunov function and MPC have been explored and illustrated. This framework is given by:

$$\inf_{u(\cdot)} \int_{t_0}^{t_0+T} (q'(x) + u^T u) dt \tag{1}$$

$$\dot{x} = f(x) + g(x)u \tag{2}$$

$$\frac{\partial V}{\partial x}(f(x) + g(x)u) \leq -\epsilon\sigma(x(t)) \tag{3}$$

$$V(x(t_0 + T)) \leq V(x_\sigma(t_0 + T)) \tag{4}$$

where u is the control law, $q'(x)$ is the cost on states, T the time horizon, $0 < \epsilon \leq 1$, $\sigma(x(t))$ is a positive definite function, $x_\sigma(t_0+T)$ is a trajectory of the system for the optimum control law obtained by solving a pointwise minimum norm optimization problem, $V(x)$ is a Control Lyapunov Function and (2) is the model of the system to be controlled [13]. The constraint (3) directly leads to the stability of MPC/CLF control scheme in Lyapunov sense, while the condition (4) preserves the requirements about the performance of the control law.

The proof of the convergence of the MPC/CLF-like approach for the mobile robot navigation purposes was given in [14]. In this paper the authors considered navigation of differentially driven mobile robots in an unknown environment using MPC/CLF scheme. The robot safety and the finite time task completeness were the core part of the proof. They introduced the Lyapunov function as the sum of a kinetic and a potential energy term, as follows,

$$V(x) = \frac{1}{2}\dot{r}^T\dot{r} + kNF(r) \tag{5}$$

where $\dot{r} = \begin{pmatrix} \dot{r}_x & \dot{r}_y \end{pmatrix}^T$ is the linear velocity of the vehicle and navigation function $NF(r)$ calculated from the given terrain. This term provides the information on the final goal position. How to construct navigation function in an unknown environment when the vehicle has only the knowledge about its sensor range environment is shown in [8]. The navigation function is computed supposing free space in unknown positions, and it is recalculated each time new information about terrain obstacles appears. Because the recalculation of navigation function happens much slower than the decision control cycle, both in [8], [14] and in our work a static terrain map is assumed. The Lyapunov function (5) has only one global minimum at $x = \begin{pmatrix} r_x & r_y & \dot{r}_x & \dot{r}_y \end{pmatrix}^T = \begin{pmatrix} x_{goal} & y_{goal} & 0 & 0 \end{pmatrix}^T$. Using such a Lyapunov function, the MPC/CLF framework has been introduced in a different form than (1-4) as follows:

$$arg \min_{u(\cdot)} V(x(t+T)) \tag{6}$$

$$\dot{x} = f(x) + g(x)u \tag{7}$$

$$\dot{V} \leq -\epsilon\sigma(x(t)) \tag{8}$$

$$\tau : [0, 1] \to C_{free}, \tau(0) = \mathbf{q}(t_0), \tau(1) = \mathbf{q}(t_0 + T) \tag{9}$$

$$\dot{r}(t_0 + T) = 0 \tag{10}$$

Let us first consider the intuitive meaning of this paradigm. Constraint (8) gives stability in the Lyapunov sense. (9) and (10) jointly guarantee the safety, where τ is the map from the initial to the final vehicle configuration **q** into the collision free space C_{free}, where **q** included position and steering angle of the vehicle. Namely, a planned collision-free trajectory ends with the vehicle standing still, which corresponds to the policy of driving a car slow enough to be able to stop it in the visible part of the road. This feature is particularly important in outdoor rough and poorly known terrain, where natural or artificial obstacles may unexpectedly appear at any time. On the other end it is quite undesirable stopping the vehicle and then restarting it at the end of every optimization cycle if not required by safety reasons.

To this purpose, the optimization procedure is repeated every time interval T_1 shorter than the optimization horizon T. This means the algorithm makes decisions every interval T_1 only among those control laws u that generate admissible velocities allowing the robot to stop safely at the end of the time interval T, and that in every successive optimization horizon the vehicle will be, at least, able to stop without hitting an obstacle, if any better control law could not be found.

The given objective function (6) has replaced the performance function (1) and the constraint (4) so that the function to be minimized is the one that gives the value of the Lyapunov function at the end of a given horizon. Considering (10) and (5) it follows that the selected function (6) gives the value of the navigation function at the end of the optimization horizon. Such optimization generates the shortest path within a MPC/CLF optimization cycle while satisfying all imposed constraints.

The dynamic model (7) is given with a differentially driven mobile robot represented by its corresponding feedback linearized model $\ddot{r} = u$ [18]. Control u was chosen in the form $u = -k\nabla NF + d$, where d is a free parameter that represents the properly chosen dissipative part of the control law in order to minimize the objective function (6) and to ensure the stability in Lyapunov sense satisfying the condition (8), and $-k\nabla NF$ is the term proportional to the gradient of NF that gives the ability of moving the vehicle in the direction of the steepest descent of this function. Since u is given in a vector form, tangential acceleration and steering angle could be extracted from such control law.

Two theorems on the final time task completeness and the safety proven in [14] makes the MPC/CLF framework especially valuable for trajectory generation of a mobile robot.

III. A NEW MPC/CLF NAVIGATION FRAMEWORK FOR MOBILE ROBOTS IN ROUGH TERRAINS

The MPC/CLF scheme (6-10) selects the optimum control law u, namely the vehicle acceleration and steering angle, among those that satisfy constraints (7-9). The selected control law provides the minimum value of the navigation function at the end of the optimization horizon. Minimizing the navigation function at the end of each optimization horizon certainly implies the fastest way to reach the goal within the given optimization framework. However, choosing the path to reach the goal as fast as possible at every decision time, may be a rather strict constraint especially when the vehicle moves in unknown rough terrains. Therefore, in this paper this condition is slightly relaxed into the following constraints (11):

$$NF(r(t_0 + T)) < NF(r(t_0 + T_1)) < NF(r(t_0)) \quad (11)$$

which, instead of pursuing the minimum value of the navigation function at the end of the optimization horizon, imposes its decrease only within each optimization cycle. Because the optimization is performed within time T while the control action u is accomplished each time T_1, both conditions of (11) need to be included. Jointly with constraint (11) that keeps the vehicle moving toward the goal since the value of the navigation function is decreasing in each optimization cycle, a new objective function can be introduced in the MPC/CLF control scheme. The new objective function proposed here considers the level of roughness of the selected path providing a new MPC/CLF scheme for the mobile robot navigation in rough terrains given with the setup (11-16):

$$\inf_{u(\cdot)} \int_r F(r)dr \quad (12)$$

$$\dot{x} = f(x) + g(x)u \quad (13)$$

$$\dot{V} \leq -\epsilon\sigma(x(t)) \quad (14)$$

$$\tau : [0, 1] \rightarrow C_{free}, \tau(0) = \mathbf{q}(t_0), \tau(1) = \mathbf{q}(t_0 + T) \quad (15)$$

$$\dot{r}(t_0 + T) = 0 \quad (16)$$

where $F(r)$ is a function that represents how hard to traverse the optimization candidate path will be. In this work the function that expresses the level of terrain roughness of a given path is approximated using the relative height of the terrain, or its deviation from flatness. This approximation is done for all candidate paths within the optimization horizon. This choice does not decrease the generality of the approach since any roughness function could be used instead. Different representations of terrain roughness were explored in [19]–[22]. In [20] and [21] the authors introduced a traversability index as a parameter describing the roughness of the terrain, while in [22] the authors used a function of the maximum velocity allowed in every point of a candidate path that still do not cause vehicle sideslip and rollover.

The local optimization illustrated in [7] and [14], based on search space generated by sampling in the control space of curvatures, is used for demonstration purposes. The MPC optimization within one time horizon was made using a discrete number of vehicle maneuvers, that is the most-left, left, straight, right and the most-right (Fig. 1), where all possible candidate paths have been evaluated at the beginning of each the MPC time horizon while searching for an optimum. In paper [23], authors have given a detailed overview of possible locally optimization techniques dealing with solution feasibility that could be also used within the proposed MPC/CLF navigation framework.

IV. ROUGH TERRAIN WITHOUT OBSTACLES OR THE KNOWN TERRAIN WITH OBSTACLES

In the following analysis of the proposed optimization scheme we assume that there is no change in the navigation function, placing the vehicle in rough terrain without obstacles or in the completely known rough terrain with obstacles.

A. Task completion

The following theorem gives the result on the maximum time vehicle needs to reach the goal position for the particular case when the vehicle moves in rough terrain without obstacles or in the completely known rough terrain with

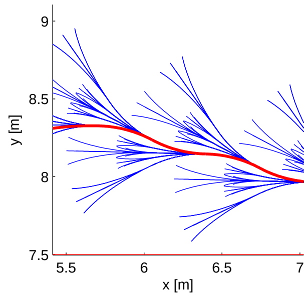
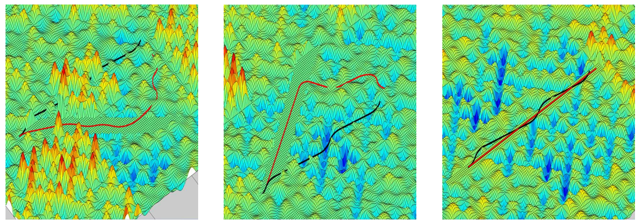


Fig. 1. Illustration of the simple optimization approach with a discrete number of maneuvers



(a) Case I: Flat section of terrain appearing on the right hand side

(b) Case II: Flat section of terrain appearing on the left hand side

(c) Case III: Narrow flat section of terrain appearing in the front

Fig. 2. Different rough terrain configurations in front of the vehicle

obstacles. These two cases do not require recalculation of the navigation function.

If $\dot{r}_{av}$ is the average velocity along the whole path, $\dot{r}_{min}$ is the smallest possible velocity of the vehicle, k and ϵ are parameters of the optimization framework, and $NF(r_0)$ the value of navigation function at the initial position of the vehicle, the resulting theorem is stated as:

***Theorem** 1: If MPC/CLF navigation scheme is used in rough terrain without obstacles (or in completely known rough terrain with obstacles) for navigation of the mobile vehicle, final time T_{goal}, and the upper time bound $T_{goal-max}$ needed for the task completion are given with:*

$$T_{goal} = \frac{1}{\epsilon \dot{r}_{av}}(kNF(r_0) + \frac{1}{2}\dot{r}_0^2)$$
$$T_{goal-max} = \frac{1}{\epsilon \dot{r}_{min}}(kNF(r_0) + \frac{1}{2}\dot{r}_0^2). \quad (17)$$

Proof: Let us first suppose that $\dot{r}(t) \neq 0$, $\forall t \in (0, T_{goal})$ along the selected path, meaning that the vehicle has not been stopped during the task.

Integration of both sides of the condition (14), for $\sigma(x(t)) = \dot{r}(t)$, given in the optimization framework along the i^{th} time horizon T_1, $t \in (t_{i-1}, t_{i-1} + T_1 = t_i)$ yields $V(t_{i-1}+T_1) - V(t_{i-1}) \leq -\varepsilon l_i$, where l_i is the arc length of the traversed path segment during the given interval. After using the expression for the control Lyapunov function, the following inequality could be obtained:

$$k(NF(r_{i-1}) - NF(r_i)) + \frac{1}{2}(\dot{r}_{i-1}^2 - \dot{r}_i^2) \geq \epsilon l_i$$
$$\Leftrightarrow k\Delta NF_i + \frac{1}{2}(\dot{r}_{i-1}^2 - \dot{r}_i^2) \geq \epsilon l_i, \quad (18)$$

where r_{i-1} and r_i are the positions at the beginning and the end of the i^{th} horizon, while ΔNF_i is the decreased value of navigation function along i^{th} path segment.

After performing the sum of both left and right sides of the last inequality along the whole path from the initial to the goal position, we obtain:

$$\sum_{i=1}^{N}(k\Delta NF_i + \frac{1}{2}(\dot{r}_{i-1}^2 - \dot{r}_i^2)) \geq \sum_{i=1}^{N} \epsilon l_i$$
$$\Leftrightarrow kNF(r_0) + \frac{1}{2}(\dot{r}_0^2 - \dot{r}_{goal}^2) \geq \epsilon l, \quad (19)$$

where l is the length of the selected path from the initial to the goal position and N is the number of traversed segments.

Since the velocity of the vehicle at the goal position is $\dot{r}_{goal} = 0$ according to MPC/CLF navigation scheme, the final inequality gives the maximum length of the MPC/CLF generated path:

$$l \leq l_{max} = \frac{1}{\epsilon}(kNF(r_0) + \frac{1}{2}\dot{r}_0^2). \quad (20)$$

Using these expressions and $l = T_{goal}\dot{r}_{av}$, we finally obtain (17).

Let us suppose now the opposite case when there is a time $t' \in (0, T_{goal})$ such that $\dot{r}(t') = 0$. This case could correspondence with the vehicle intention to climb the navigation surface, meaning that the condition (14) is not satisfied. In this case the algorithm is made such that the vehicle chooses a control pair to turn toward the steepest descent of navigation function and starts moving. This control law will decrease the value of navigation function more than any control pair of the hypothetically worst case considered above (for the case $\dot{r}(t') \neq 0, \forall t \in (0, T_{goal})$) when the small possible decrease of the navigation value is performed (yet not stopping the vehicle).

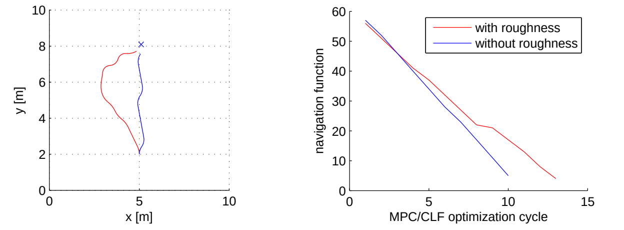
This means that the worst case scenario is executed without stopping the vehicle, that is $\dot{r}(t) \neq 0, \forall t \in (0, T_{goal})$, while decreasing navigation function as small as possible yielding the least possible value of $\dot{r}_{av}$ (In practical case, a threshold velocity value could be imposed considering all values beyond it equal to zero). This implies that $\forall \dot{r}(t), t \in (0, T_{goal})$ the upper time bound is given by (17). ■

B. The bounds of the possible path

***Corollary** 1: If MPC/CLF navigation scheme is used in rough terrain without obstacles (or in completely known rough terrain with obstacles) for navigation of the mobile vehicle started with zero velocity, the bounds of the possible length of the selected path needed for the task completion are given by:*

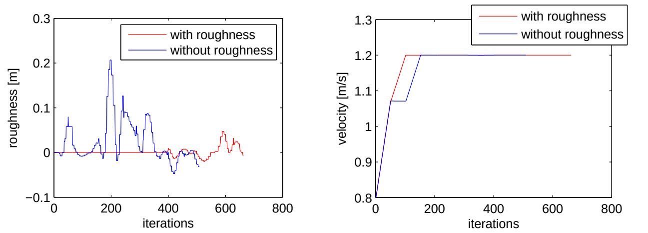
$$NF(r_0) \leq l \leq \frac{k}{\epsilon}NF(r_0). \quad (21)$$

The proof directly follows from (20) and the fact that the value of navigation function at initial position $NF(r_0)$ describes the shortest possible path to the goal.



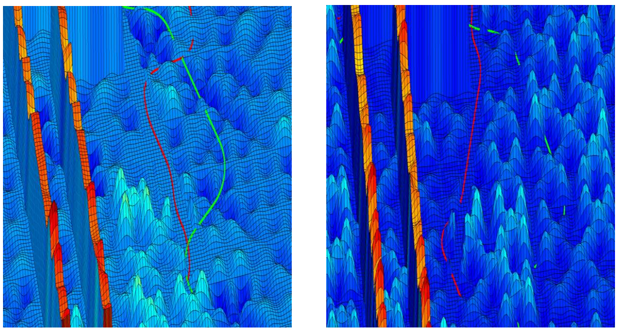
(a) Case I: Paths that have been selected (b) Case I: Value of navigation function at the end of optimization cycle along the selected path

Fig. 3.



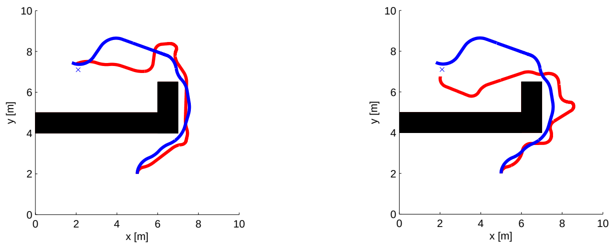
(a) Case I: Terrain roughness along the selected paths (b) Case I: Vehicle velocity diagram

Fig. 4.



(a) Case IV: Rough terrain with obstacle I (b) Case V: Rough terrain with obstacle II

Fig. 5. Different rough terrain configurations with obstacles

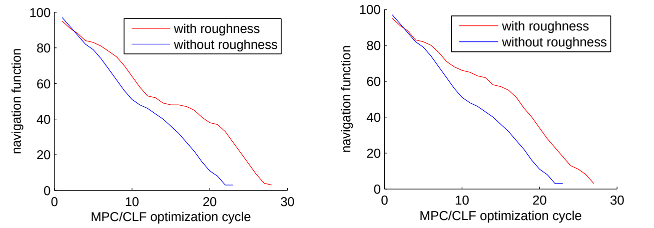


(a) Case IV (b) Case V

Fig. 6. Paths that have been selected

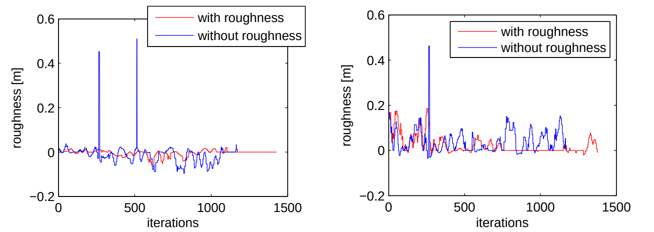
V. SIMULATION RESULTS

In the following simulations a differentially driven mobile vehicle used in [2] and [14] is considered. It was assumed that the velocity and the terrain roughness are such that



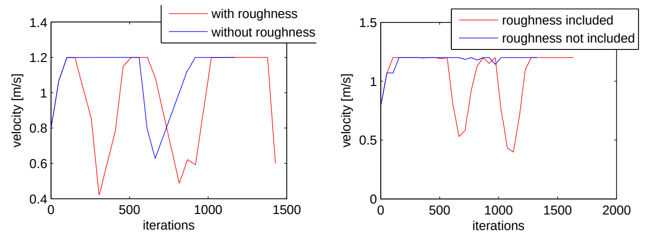
(a) Case IV (b) Case V

Fig. 7. Value of navigation function at the end of optimization cycle along the selected path



(a) Case IV (b) Case V

Fig. 8. Terrain roughness along the selected paths



(a) Case IV (b) Case V

Fig. 9. Vehicle velocity diagram

do not cause sideslip and rollover of the vehicle. Furthermore, since the simplified planar model is used to generate reference trajectories to the vehicle, terrain roughness was considered as disturbance to the trajectory tracking algorithm. The error caused by the tracking is eliminated at the beginning of each optimization cycle thanks to the MPC paradigm where the observed states are periodically updated. The simulation itself is organized within two main nominal terrain configurations with and without obstacles. Roughness that was additionally imposed to these nominal structures was generated using the fractal features of a natural terrain according to the nice algorithm illustrated in [11]. Roughness is not considered as an obstacle but as an unevenness of the given terrain that could be traversed by the vehicle with different levels of hardness (Fig. 2 and Fig. 5).

The main idea of the simulation using only terrain roughness but no obstacles is to present the possibility of using the MPC/CLF navigation framework and to compare it to the one made particularly for flat terrains. In order to illustrate the possible usefulness of the approach some flat terrains with different roughness were generated where all terrains had a section of a flat path in the three different directions (Fig. 2) along which the value of navigation function decreased. It has been shown that if there is a flat terrain section (or less rough terrain for the current horizon) in front of the vehicle toward the goal position, the vehicle tends to follow this section as long as the navigation function decreases. For the first case, Fig. 3(a) and Fig. 3(b) depict the generated paths and the decrease of navigation function of the algorithm that consider terrain roughness and the one that does not take it into the optimization process. The decrease of navigation function ensures that the vehicle approaches the goal or the next waypoint given by the operator. In Fig. 4(a) a comparison between morphology of the terrain that has been traversed by the vehicle using the two algorithms is illustrated. It could be visually concluded that the vehicle with the proposed scheme traverses less roughness than in the case when roughness is not considered, particularly because the vehicle has been continuously driven along the flat part of the terrain as long as it was possible. Fig. 4(b) shows the velocity diagram for both cases.

Other two examples of the given flat terrain sections are shown in Fig. 2(b) and Fig. 2(c) that confirm vehicle capability to reach the goal while choosing more appropriate terrain areas in the sense of the level of roughness. Namely, MPC/CLF algorithm generated paths over the flat terrain while still keeping the decrease of navigation function.

The main idea of the second terrain configuration with an obstacle was to illustrate the potential of the algorithm to reach the goal while avoiding collisions and still keeping the flat terrain parts along which navigation function value decreases. This is demonstrated within two different types of roughness (Fig. 5(a) and 5(b)) where the vehicle has shown the ability to drive along the flat terrain areas if they existed on its way to the goal and if the vehicle connects to these areas while completing the task. Fig. 6, 7, 8 and 9, as in the previous example, represent the chosen path, navigation function, the roughness that has been traversed and the velocity diagram, respectively.

VI. Conclusion

This work deals with motion planning in natural terrains and environments. The Model Predictive Control combined with the Control Lyapunov Function optimization framework has been reviewed and its extension to the trajectory planning of mobile robots in outdoor natural terrain has been proposed. Some theoretical modifications have been introduced to MPC/CLF navigation scheme and it has been shown by simulation that the resulting optimization scheme is truly effective to the purpose, as the vehicle was able to find the smoothest paths, in rough terrains. The algorithm gives the trajectory satisfying constraints on obstacle avoidance, decreasing the value of navigation function along the path, and minimizing the level of roughness of the path to be traversed. The minimization of the function that describes the terrain roughness could significantly relax the effort the vehicle deals with while approaching the goal position. Traversing the less rough terrain sections helps the vehicle to increase its ability for high-speed maneuvers that do not cause unwanted effects such as sideslip and rollover.

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